Mathematica 11.3 Integration Test Results

Test results for the 178 problems in "5.6.1 u (a+b arccsc(c x))^n.m"

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a + b \operatorname{ArcCsc}[c x]\right)^3 \, dx$$

Optimal (type 4, 220 leaves, 11 steps):

$$\frac{b^2 x \left(a + b \operatorname{ArcCsc}[c x]\right)}{c^2} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}}\, x^2 \left(a + b \operatorname{ArcCsc}[c x]\right)^2}{2 c} + \frac{1}{3} x^3 \left(a + b \operatorname{ArcCsc}[c x]\right)^3 + \frac{b \left(a + b \operatorname{ArcCsc}[c x]\right)^2 \operatorname{ArcTanh}\left[e^{i \operatorname{ArcCsc}[c x]}\right]}{c^3} + \frac{b^3 \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{c^2 x^2}}\right]}{c^3} - \frac{i b^2 \left(a + b \operatorname{ArcCsc}[c x]\right) \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcCsc}[c x]}\right]}{c^3} + \frac{i b^2 \left(a + b \operatorname{ArcCsc}[c x]\right) \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcCsc}[c x]}\right]}{c^3} + \frac{b^3 \operatorname{PolyLog}\left[3, -e^{i \operatorname{ArcCsc}[c x]}\right]}{c^3} - \frac{b^3 \operatorname{PolyLog}\left[3, e^{i \operatorname{ArcCsc}[c x]}\right]}{c^3}$$

Result (type 4, 580 leaves):

$$\frac{a^3 x^3}{3}+\frac{a^2 b x^2 \sqrt{\frac{-1+c^2 x^2}{c^2 x^2}}}{2 c}+a^2 b x^3 \text{ArcCsc}[c x]+$$

$$\frac{a^2 b \text{Log}\left[x \left(1+\sqrt{\frac{-1+c^2 x^2}{c^2 x^2}}\right)\right]}{2 c^3}+\frac{1}{8 c^3} a b^2 \left(-8 \text{i} \text{PolyLog}\left[2,-e^{\text{i} \text{ArcCsc}[c x]}\right]+\right.$$

$$2 c^3 x^3 \left(2+4 \text{ArcCsc}[c x]^2-2 \text{Cos}[2 \text{ArcCsc}[c x]]-\frac{3 \text{ArcCsc}[c x] \text{Log}\left[1-e^{\text{i} \text{ArcCsc}[c x]}\right]}{c x}+\right.$$

$$\frac{3 \text{ArcCsc}[c x] \text{Log}\left[1+e^{\text{i} \text{ArcCsc}[c x]}\right]}{c x}+\frac{4 \text{i} \text{PolyLog}\left[2,e^{\text{i} \text{ArcCsc}[c x]}\right]}{c^3 x^3}+$$

$$2 \text{ArcCsc}[c x] \text{Sin}[2 \text{ArcCsc}[c x]]+\text{ArcCsc}[c x] \text{Log}\left[1-e^{\text{i} \text{ArcCsc}[c x]}\right] \text{Sin}[3 \text{ArcCsc}[c x]]-$$

$$\left.\left.\text{ArcCsc}[c x] \text{Log}\left[1+e^{\text{i} \text{ArcCsc}[c x]}\right] \text{Sin}[3 \text{ArcCsc}[c x]]\right)\right)+$$

$$\frac{1}{48 c^3} b^3 \left(24 \text{ArcCsc}[c x] \text{Cot}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]+4 \text{ArcCsc}[c x]^3 \text{Cot}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]+\right.$$

$$6 \text{ArcCsc}[c x]^2 \text{Csc}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^2+\frac{\text{ArcCsc}[c x]^3 \text{Csc}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^4}{c x}-$$

$$24 \text{ArcCsc}[c x]^2 \text{Log}\left[1-e^{\text{i} \text{ArcCsc}[c x]}\right]+24 \text{ArcCsc}[c x]^2 \text{Log}\left[1+e^{\text{i} \text{ArcCsc}[c x]}\right]-$$

$$48 \text{Log}\left[\text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right]-48 \text{i} \text{ArcCsc}[c x] \text{PolyLog}\left[2,-e^{\text{i} \text{ArcCsc}[c x]}\right]+$$

$$48 \text{i} \text{ArcCsc}[c x] \text{PolyLog}\left[2,e^{\text{i} \text{ArcCsc}[c x]}\right]+48 \text{PolyLog}\left[3,-e^{\text{i} \text{ArcCsc}[c x]}\right]-$$

$$48 \text{PolyLog}\left[3,e^{\text{i} \text{ArcCsc}[c x]}\right]-6 \text{ArcCsc}[c x]^2 \text{Sec}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^2+$$

$$16 c^3 x^3 \text{ArcCsc}[c x]^3 \text{Sin}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^4+$$

$$\left.24 \text{ArcCsc}[c x] \text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]+4 \text{ArcCsc}[c x]^3 \text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{d+e x} \left(a+b \text{ArcCsc}[c x]\right) dx$$

Optimal (type 4, 496 leaves, 31 steps):

$$\frac{4\,b\,d\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{105\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}-\frac{4\,b\,\left(d+e\,x\right)^{3/2}\,\left(1-c^2\,x^2\right)}{35\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^3}-$$

$$\frac{4\,d\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^3}+\frac{2\,\left(d+e\,x\right)^{7/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{7\,e^3}+$$

$$\left(4\,b\,\left(5\,c^2\,d^2-9\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(105\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,\left(d+e\,x\right)}{c\,d+e}}\right)-$$

$$\left(4\,b\,d\,\left(9\,c^2\,d^2-e^2\right)\,\sqrt{\frac{c\,\left(d+e\,x\right)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(105\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-$$

$$\left(32\,b\,d^4\,\sqrt{\frac{c\,\left(d+e\,x\right)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(105\,c\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 428 leaves) :

$$\frac{1}{105\,e^3}2\left(\frac{2\,b\,e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+e\,x}\,\left(2\,d+3\,e\,x\right)}{c}+a\sqrt{d+e\,x}\,\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)+\right.$$
$$b\sqrt{d+e\,x}\,\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)\text{ArcCsc}[c\,x]-$$
$$\left(2\,i\,b\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\left(\left(5\,c^3\,d^3-5\,c^2\,d^2\,e-9\,c\,d\,e^2+9\,e^3\right)\text{EllipticE}\left[\right.\right.\right.$$
$$\left.i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+\left(4\,c^3\,d^3+5\,c^2\,d^2\,e+8\,c\,d\,e^2-9\,e^3\right)$$
$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-8\,c^3\,d^3\,\text{EllipticPi}\left[1+\frac{e}{c\,d},\right.$$
$$\left.\left.\left.\left.i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Big/\left(c^4\sqrt{-\frac{c}{c\,d+e}}\sqrt{1-\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int x\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,dx$$

Optimal (type 4, 404 leaves, 24 steps):

$$-\frac{4 b \sqrt{d+e x}\,\left(1-c^2 x^2\right)}{15 c^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x}-\frac{2 d\,(d+e x)^{3/2}\,(a+b \operatorname{ArcCsc}[c x])}{3 e^2}+$$

$$\frac{2\,(d+e x)^{5/2}\,(a+b \operatorname{ArcCsc}[c x])}{5 e^2}-\frac{8 b d \sqrt{d+e x}\,\sqrt{1-c^2 x^2}\, \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{15 c^2 e \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}+$$

$$\left(4 b\left(3 c^2 d^2-e^2\right) \sqrt{\frac{c\,(d+e x)}{c d+e}}\, \sqrt{1-c^2 x^2}\, \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]\right)\Big/$$

$$\left(15 c^4 e \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)+$$

$$\frac{8 b d^3 \sqrt{\frac{c (d+e x)}{c d+e}}\, \sqrt{1-c^2 x^2}\, \operatorname{EllipticPi}\left[2,\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{15 c e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 368 leaves):

$$\frac{1}{15}\left(\frac{4 b \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}{c}+\right.$$

$$\frac{2 a \sqrt{d+e x}\,\left(-2 d^2+d e x+3 e^2 x^2\right)}{e^2}+\frac{2 b \sqrt{d+e x}\,\left(-2 d^2+d e x+3 e^2 x^2\right) \operatorname{ArcCsc}[c x]}{e^2}-$$

$$\left(4 i b \sqrt{\frac{e\,(1+c x)}{-c d+e}}\, \sqrt{\frac{e-c e x}{c d+e}}\, \left(-2 c d\,(c d-e)\, \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}}\, \sqrt{d+e x}\right],\right.\right.\right.$$

$$\left.\frac{c d+e}{c d-e}\right]+\left(-c^2 d^2-2 c d e+e^2\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}}\, \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]+$$

$$\left.\left.2 c^2 d^2 \operatorname{EllipticPi}\left[1+\frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}}\, \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/$$

$$\left.\left(c^3 e^2 \sqrt{-\frac{c}{c d+e}}\, \sqrt{1-\frac{1}{c^2 x^2}}\, x\right)\right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{d+e x}\,(a+b\,\text{ArcCsc}[c\,x])\,dx$$

Optimal (type 4, 315 leaves, 15 steps):

$$\frac{2\,(d+e x)^{3/2}\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e}-\frac{4\,b\,\sqrt{d+e x}\,\sqrt{1-c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{\frac{c\,(d+e x)}{c\,d+e}}}-$$

$$\frac{4\,b\,d\,\sqrt{\frac{c\,(d+e x)}{c\,d+e}}\,\sqrt{1-c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{d+e x}}-$$

$$\frac{4\,b\,d^2\,\sqrt{\frac{c\,(d+e x)}{c\,d+e}}\,\sqrt{1-c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{d+e x}}$$

Result (type 4, 275 leaves):

$$\frac{1}{3\,e}2\left(a\,(d+e x)^{3/2}+b\,(d+e x)^{3/2}\,\text{ArcCsc}[c\,x]+\left(2\,i\,b\,\sqrt{\frac{e\,(1+c x)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\left((c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e x}\right],\frac{c\,d+e}{c\,d-e}\right]+(-2\,c\,d+e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e x}\right],\frac{c\,d+e}{c\,d-e}\right]+c\,d\,\text{EllipticPi}\left[1+\frac{e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Bigg/\left(c^2\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\right)\right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^{3/2}\,(a+b\,\text{ArcCsc}[c\,x])\,dx$$

Optimal (type 4, 372 leaves, 22 steps):

$$-\frac{4\,b\,e\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{15\,c^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,(d+e\,x)^{5/2}\,(a+b\,\text{ArcCsc}[c\,x])}{5\,e}-$$

$$\frac{28\,b\,d\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-$$

$$\left(4\,b\,\left(2\,c^2\,d^2+e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(15\,c^4\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-\frac{4\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{5\,c\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 333 leaves):

$$\frac{1}{15}\left(\frac{4\,b\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}{c}+\frac{6\,a\,(d+e\,x)^{5/2}}{e}+\right.$$

$$\frac{6\,b\,(d+e\,x)^{5/2}\,\text{ArcCsc}[c\,x]}{e}-\left(4\,\text{i}\,b\,\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\right.$$

$$\left(-7\,c\,d\,(c\,d-e)\,\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+\left(9\,c^2\,d^2-7\,c\,d\,e+e^2\right)\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-3\,c^2\,d^2\,\text{EllipticPi}\left[1+\frac{e}{c\,d},\right.$$

$$\left.\left.\left.\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Big/\left(c^3\,e\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 714 leaves, 27 steps):

$$-\frac{4 b \sqrt{d+e x}\left(1-c^2 x^2\right)}{35 c^3 e \sqrt{1-\frac{1}{c^2 x^2}}}+\frac{4 b d \sqrt{d+e x}\left(1-c^2 x^2\right)}{21 c^3 e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x}-\frac{2 d^3 \sqrt{d+e x}\,(a+b \operatorname{ArcCsc}[c x])}{e^4}+$$

$$\frac{2 d^2 (d+e x)^{3/2}(a+b \operatorname{ArcCsc}[c x])}{e^4}-\frac{6 d (d+e x)^{5/2}(a+b \operatorname{ArcCsc}[c x])}{5 e^4}+$$

$$\frac{2 (d+e x)^{7/2}(a+b \operatorname{ArcCsc}[c x])}{7 e^4}-\frac{24 b d^2 \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{35 c^2 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}+$$

$$\left(4 b \left(2 c^2 d^2-9 e^2\right) \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]\right) \Big/$$

$$\left(105 c^4 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}\right)+$$

$$\frac{64 b d^3 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{35 c^2 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-$$

$$\left(32 b d (c d-e)(c d+e) \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]\right) \Big/$$

$$\left(105 c^4 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)+$$

$$\frac{64 b d^4 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{35 c e^4 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 429 leaves) :

$$\frac{1}{105\,e^4}\,2\left(\frac{2\,b\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\,\left(-5\,d+3\,e\,x\right)}{c}+3\,a\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)+\right.$$

$$3\,b\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)\,\text{ArcCsc}[c\,x]-$$

$$\left(2\,i\,b\,\sqrt{\frac{e\,\left(1+c\,x\right)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\left(\left(-16\,c^3\,d^3+16\,c^2\,d^2\,e-9\,c\,d\,e^2+9\,e^3\right)\,\text{EllipticE}\right[\right.$$

$$i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\Big]+\left(-24\,c^3\,d^3-16\,c^2\,d^2\,e+c\,d\,e^2-9\,e^3\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+48\,c^3\,d^3\,\text{EllipticPi}\Big[$$

$$\left.\left.\left.1+\frac{e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Bigg/\left(c^4\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 530 leaves, 20 steps):

$$-\frac{4\,b\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{15\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^3}-\frac{4\,d\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^3}+$$

$$\frac{2\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^3}+\frac{4\,b\,d\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{5\,c^2\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-$$

$$\frac{32\,b\,d^2\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^2\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\left(4\,b\,\left(c\,d-e\right)\,\left(c\,d+e\right)\,\sqrt{\frac{c\,\left(d+e\,x\right)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(15\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-$$

$$\frac{32\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 365 leaves) :

$$\frac{1}{15 e^3} 2 \left(\frac{2 b e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}{c} + a \sqrt{d+e x} \left(8 d^2 - 4 d e x + 3 e^2 x^2\right) + b \sqrt{d+e x} \left(8 d^2 - 4 d e x + 3 e^2 x^2\right) \text{ArcCsc}[c x] - \left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e - c e x}{c d+e}} \left(3 c d (c d - e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] + \left(4 c^2 d^2 + 3 c d e + e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] - 8 c^2 d^2 \text{EllipticPi}\left[1+\frac{e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right]\right)\right) \Big/ \left(c^3 \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}}\, x\right) \right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \left(a + b \text{ArcCsc}[c x]\right)}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$-\frac{2 d \sqrt{d+e x} \left(a+b \text{ArcCsc}[c x]\right)}{e^2} + \frac{2 (d+e x)^{3/2} \left(a+b \text{ArcCsc}[c x]\right)}{3 e^2} - \frac{4 b \sqrt{d+e x} \sqrt{1-c^2 x^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c^2 e \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}} + \frac{8 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c^2 e \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}} + \frac{8 b d^2 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 289 leaves):

$$\frac{1}{3 e^2}2 \left(a (-2 d+e x) \sqrt{d+e x}+b (-2 d+e x) \sqrt{d+e x} \text{ArcCsc}[c x]+\left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left((c d-e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]+(c d+e) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-2 c d \text{EllipticPi}\left[1+\frac{e}{c d},i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(c^2 \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}} x\right)\right)$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \text{ArcCsc}[c x]}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 212 leaves, 9 steps):

$$\frac{2 \sqrt{d+e x} (a+b \text{ArcCsc}[c x])}{e}-\frac{4 b \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{c^2 \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}-\frac{4 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{c e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 212 leaves):

$$\frac{1}{e}2 \left(a \sqrt{d+e x}+b \sqrt{d+e x} \text{ArcCsc}[c x]-\left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-\text{EllipticPi}\left[1+\frac{e}{c d},i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(c \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}} x\right)\right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 (a+b \text{ArcCsc}[c x])}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 551 leaves, 23 steps):

$$-\frac{4 b \sqrt{d+e x} \left(1-c^2 x^2\right)}{15 c^3 e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x}+\frac{2 d^3 \left(a+b \operatorname{ArcCsc}[c x]\right)}{e^4 \sqrt{d+e x}}+\frac{6 d^2 \sqrt{d+e x} \left(a+b \operatorname{ArcCsc}[c x]\right)}{e^4}-$$

$$\frac{2 d (d+e x)^{3/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{e^4}+\frac{2 (d+e x)^{5/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{5 e^4}+$$

$$\frac{32 b d \sqrt{d+e x} \sqrt{1-c^2 x^2}\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 c^2 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}-$$

$$\frac{8 b d^2 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{c^2 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-$$

$$\left(4 b \left(2 c^2 d^2+e^2\right) \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]\right) \Big/$$

$$\left(15 c^4 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)-$$

$$\frac{64 b d^3 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\ \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{5 c e^4 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 387 leaves):

$$\frac{1}{15 e^4}2\left(\frac{2 b e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}{c}+\frac{3 a \left(16 d^3+8 d^2 e x-2 d e^2 x^2+e^3 x^3\right)}{\sqrt{d+e x}}+\frac{3 b \left(16 d^3+8 d^2 e x-2 d e^2 x^2+e^3 x^3\right) \text{ArcCsc}[c x]}{\sqrt{d+e x}}-\left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left(8 c d (c d-e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]+\left(24 c^2 d^2+8 c d e+e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-48 c^2 d^2 \text{EllipticPi}\left[1+\frac{e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(c^3 \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}}\, x\right)\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a+b \text{ArcCsc}[c x])}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 369 leaves, 16 steps):

$$-\frac{2 d^2 (a+b \text{ArcCsc}[c x])}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x}\, (a+b \text{ArcCsc}[c x])}{e^3}+\frac{2 (d+e x)^{3/2} (a+b \text{ArcCsc}[c x])}{3 e^3}-\frac{4 b \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 c^2 e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}+\frac{20 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 c^2 e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}+\frac{32 b d^2 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 c e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 312 leaves):

$$\frac{1}{3 e^3} 2 \left(\frac{a \left(-8 d^2-4 d e x+e^2 x^2\right)}{\sqrt{d+e x}}+\frac{b \left(-8 d^2-4 d e x+e^2 x^2\right) \text{ArcCsc}[c x]}{\sqrt{d+e x}}-\left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left((-c d+e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-(4 c d+e) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]+8 c d \text{EllipticPi}\left[1+\frac{e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(c^2 \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}}\, x\right)\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x (a+b \text{ArcCsc}[c x])}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 238 leaves, 11 steps):

$$\frac{2 d (a+b \text{ArcCsc}[c x])}{e^2 \sqrt{d+e x}}+\frac{2 \sqrt{d+e x} (a+b \text{ArcCsc}[c x])}{e^2}-\frac{4 b \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{c^2 e \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-\frac{8 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{c e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 226 leaves):

$$\frac{1}{e^2}2\left(\frac{a\,(2\,d+e\,x)}{\sqrt{d+e\,x}}+\frac{b\,(2\,d+e\,x)\,\text{ArcCsc}[c\,x]}{\sqrt{d+e\,x}}-\left(2\,\mathrm{i}\,b\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\left(\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-2\,\text{EllipticPi}\left[1+\frac{e}{c\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Big/\left(c\sqrt{-\frac{c}{c\,d+e}}\sqrt{1-\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 602 leaves, 31 steps):

$$-\frac{4 b d^2 \left(1-c^2 x^2\right)}{3 c e^2 \left(c^2 d^2-e^2\right) \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}+\frac{2 d^3 (a+b \text{ArcCsc}[c x])}{3 e^4 (d+e x)^{3/2}}-$$

$$\frac{6 d^2 (a+b \text{ArcCsc}[c x])}{e^4 \sqrt{d+e x}}-\frac{6 d \sqrt{d+e x}\, (a+b \text{ArcCsc}[c x])}{e^4}+\frac{2 (d+e x)^{3/2} (a+b \text{ArcCsc}[c x])}{3 e^4}+$$

$$\frac{8 b d^2 \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 e^3 \left(c^2 d^2-e^2\right) \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}-$$

$$\left(4 b \left(2 c^2 d^2-e^2\right) \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]\right)\Big/$$

$$\left(3 c^2 e^3 \left(c^2 d^2-e^2\right) \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}\right)+$$

$$\frac{32 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 c^2 e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}+$$

$$\frac{64 b d^2 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{3 c e^4 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 398 leaves) :

$$\frac{1}{3 e^4} 2 \left(\frac{2 b c d^2 e^2 \sqrt{1 - \frac{1}{c^2 x^2}}\, x}{(c^2 d^2 - e^2) \sqrt{d + e x}} + \frac{a (-16 d^3 - 24 d^2 e x - 6 d e^2 x^2 + e^3 x^3)}{(d + e x)^{3/2}} + \frac{b (-16 d^3 - 24 d^2 e x - 6 d e^2 x^2 + e^3 x^3) \text{ArcCsc}[c x]}{(d + e x)^{3/2}} + \frac{1}{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}\, x} 2 i b \sqrt{-\frac{c}{c d + e}} \sqrt{\frac{e - c e x}{c d + e}} \sqrt{-\frac{e + c e x}{c d - e}} \left(e^2 \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right] - (8 c^2 d^2 + 8 c d e + e^2) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right] + 16 c d (c d + e) \text{EllipticPi}\left[1 + \frac{e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right] \right) \right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a + b \text{ArcCsc}[c x])}{(d + e x)^{5/2}} \, dx$$

Optimal (type 4, 440 leaves, 25 steps):

$$\frac{4 b d (1 - c^2 x^2)}{3 c e (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{d + e x}} - \frac{2 d^2 (a + b \text{ArcCsc}[c x])}{3 e^3 (d + e x)^{3/2}} + \frac{4 d (a + b \text{ArcCsc}[c x])}{e^3 \sqrt{d + e x}} + \frac{2 \sqrt{d + e x} (a + b \text{ArcCsc}[c x])}{e^3} - \frac{4 b d \sqrt{d + e x} \sqrt{1 - c^2 x^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{3 e^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d + e x)}{c d + e}}} - \frac{4 b \sqrt{\frac{c (d + e x)}{c d + e}} \sqrt{1 - c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{c^2 e^2 \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{d + e x}} - \frac{32 b d \sqrt{\frac{c (d + e x)}{c d + e}} \sqrt{1 - c^2 x^2} \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{3 c e^3 \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{d + e x}}$$

Result (type 4, 367 leaves):

$$\frac{2}{3}\left(\frac{2\,b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{\left(-c^2\,d^2\,e+e^3\right)\,\sqrt{d+e\,x}}+\frac{a\,\left(8\,d^2+12\,d\,e\,x+3\,e^2\,x^2\right)}{e^3\,\left(d+e\,x\right)^{3/2}}+\frac{b\,\left(8\,d^2+12\,d\,e\,x+3\,e^2\,x^2\right)\,\text{ArcCsc}[c\,x]}{e^3\,\left(d+e\,x\right)^{3/2}}-\left(2\,\text{i}\,b\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\sqrt{-\frac{e+c\,e\,x}{c\,d-e}}\,\left(c\,d\,\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-\left(4\,c\,d+3\,e\right)\,\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+8\,\left(c\,d+e\right)\,\text{EllipticPi}\left[1+\frac{e}{c\,d},\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Big/\left(c^2\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 314 leaves, 19 steps):

$$-\frac{4\,b\,\left(1-c^2\,x^2\right)}{3\,c\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+\frac{2\,d\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^2\,\left(d+e\,x\right)^{3/2}}-\frac{2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2\,\sqrt{d+e\,x}}+\frac{4\,b\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,e\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+\frac{8\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 345 leaves):

$$\frac{4\,b\,c\,\sqrt{1-\frac{1}{c^2 x^2}}\;x}{3\,\left(c^2 d^2-e^2\right)\sqrt{d+e\,x}}-\frac{2\,a\,\left(2\,d+3\,e\,x\right)}{3\,e^2\,\left(d+e\,x\right)^{3/2}}-$$

$$\frac{2\,b\,\left(2\,d+3\,e\,x\right)\,\text{ArcCsc}[c\,x]}{3\,e^2\,\left(d+e\,x\right)^{3/2}}+\Bigg(4\,i\,b\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{\frac{e\,\left(1+c\,x\right)}{-c\,d+e}}$$

$$\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\Bigg(c\,d\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\Big],\frac{c\,d+e}{c\,d-e}\Big]-$$

$$c\,d\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\Big],\frac{c\,d+e}{c\,d-e}\Big]+2\,\left(c\,d+e\right)$$

$$\text{EllipticPi}\Big[1+\frac{e}{c\,d},\,i\,\text{ArcSinh}\Big[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\Big],\frac{c\,d+e}{c\,d-e}\Big]\Bigg)\Bigg)\Bigg/\left(3\,c^2\,d\,e^2\,\sqrt{1-\frac{1}{c^2 x^2}}\;x\right)$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 298 leaves, 12 steps):

$$\frac{4\,b\,e\,\left(1-c^2 x^2\right)}{3\,c\,d\,\left(c^2 d^2-e^2\right)\sqrt{1-\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e\,\left(d+e\,x\right)^{3/2}}-$$

$$\frac{4\,b\,\sqrt{d+e\,x}\,\sqrt{1-c^2 x^2}\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\Big],\frac{2\,e}{c\,d+e}\Big]}{3\,d\,\left(c^2 d^2-e^2\right)\sqrt{1-\frac{1}{c^2 x^2}}\;x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+$$

$$\frac{4\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2 x^2}\,\text{EllipticPi}\Big[2,\text{ArcSin}\Big[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\Big],\frac{2\,e}{c\,d+e}\Big]}{3\,c\,d\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}}$$

Result (type 4, 326 leaves):

$$\frac{1}{3e}2\left(-\frac{a}{(d+ex)^{3/2}}-\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d^3-de^2)\sqrt{d+ex}}-\frac{b\,\text{ArcCsc}[cx]}{(d+ex)^{3/2}}+\right.$$

$$\left(2\,i\,b\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-cd\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right],\frac{cd+e}{cd-e}\right]+\right.\right.$$

$$cd\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right],\frac{cd+e}{cd-e}\right]+$$

$$\left.\left.(cd+e)\,\text{EllipticPi}\left[1+\frac{e}{cd},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right],\frac{cd+e}{cd-e}\right]\right)\right)\Big/$$

$$\left.\left(d^2\left(-\frac{c}{cd+e}\right)^{3/2}(cd+e)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\right)$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsc}[cx]}{(d+ex)^{7/2}}\,dx$$

Optimal (type 4, 540 leaves, 19 steps):

$$\frac{4\,b\,e\,\left(1-c^2\,x^2\right)}{15\,c\,d\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\left(d+e\,x\right)^{3/2}}+\frac{16\,b\,c\,e\,\left(1-c^2\,x^2\right)}{15\,\left(c^2\,d^2-e^2\right)^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,e\,\left(1-c^2\,x^2\right)}{5\,c\,d^2\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e\,\left(d+e\,x\right)^{5/2}}-$$

$$\left(4\,b\,\left(7\,c^2\,d^2-3\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{c\,d+e}\right]\right)\Big/$$

$$\left(15\,\left(c^2\,d^3-d\,e^2\right)^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{\frac{c\,\left(d+e\,x\right)}{c\,d+e}}\right)+$$

$$\frac{4\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{c\,d+e}\right]}{15\,d\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\;\text{EllipticPi}\left[2,\,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{c\,d+e}\right]}{5\,c\,d^2\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}$$

Result (type 4, 407 leaves) :

$$\frac{1}{15 e}2\left(-\frac{3 a}{(d+e x)^{5/2}}-\frac{2 b c e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \left(-e^2 (4 d+3 e x)+c^2 d^2 (8 d+7 e x)\right)}{\left(c^2 d^3-d e^2\right)^2 (d+e x)^{3/2}}-\frac{3 b \text{ArcCsc}[c x]}{(d+e x)^{5/2}}-\left(2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left(c d \left(7 c^2 d^2-3 e^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-c d \left(6 c^2 d^2-c d e-3 e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]-3 (c d-e) (c d+e)^2 \text{EllipticPi}\left[1+\frac{e}{c d},i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(d^3 (c d-e) \left(-\frac{c}{c d+e}\right)^{3/2} (c d+e)^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x\right)\right)$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a+b \text{ArcCsc}[c x])}{d+e x^2}\, dx$$

Optimal (type 4, 565 leaves, 25 steps):

$$\frac{x\,(a+b\,\mathrm{ArcCsc}[c\,x])}{e}+\frac{b\,\mathrm{ArcTanh}\left[\sqrt{1-\frac{1}{c^2 x^2}}\right]}{c\,e}-\frac{\sqrt{-d}\,(a+b\,\mathrm{ArcCsc}[c\,x])\,\mathrm{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{\sqrt{-d}\,(a+b\,\mathrm{ArcCsc}[c\,x])\,\mathrm{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-$$

$$\frac{\sqrt{-d}\,(a+b\,\mathrm{ArcCsc}[c\,x])\,\mathrm{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{\sqrt{-d}\,(a+b\,\mathrm{ArcCsc}[c\,x])\,\mathrm{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-$$

$$\frac{i\,b\,\sqrt{-d}\,\mathrm{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+\frac{i\,b\,\sqrt{-d}\,\mathrm{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-$$

$$\frac{i\,b\,\sqrt{-d}\,\mathrm{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+\frac{i\,b\,\sqrt{-d}\,\mathrm{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\mathrm{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}$$

Result (type 4, 1260 leaves):

$$\frac{1}{4\,c\,e^{3/2}}\,i\left(-4\,i\,a\,c\,\sqrt{e}\,x-4\,i\,b\,c\,\sqrt{e}\,x\,\mathrm{ArcCsc}[c\,x]+4\,i\,a\,c\,\sqrt{d}\,\mathrm{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$

$$8\,i\,b\,c\,\sqrt{d}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\,(\pi+2\,\mathrm{ArcCsc}[c\,x])\right]}{\sqrt{c^2 d+e}}\right]-$$

$$8\,i\,b\,c\,\sqrt{d}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\,(\pi+2\,\mathrm{ArcCsc}[c\,x])\right]}{\sqrt{c^2 d+e}}\right]+$$

$$b\,c\,\sqrt{d}\,\pi\,\mathrm{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\mathrm{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,c\,\sqrt{d}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\mathrm{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,c\,\sqrt{d}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\mathrm{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$b\,c\,\sqrt{d}\,\pi\,\mathrm{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\mathrm{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,b\,c\,\sqrt{d}\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,b\,c\,\sqrt{d}\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,c\,\sqrt{d}\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]-b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]-4\,i\,b\,\sqrt{e}\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right]+$$

$$4\,i\,b\,\sqrt{e}\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right]+2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]\right)$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int \frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 507 leaves, 26 steps):

$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e}+\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e}+$$
$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e}+\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e}-$$
$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e}-$$
$$\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e}-$$
$$\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e}+\frac{i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{2\,e}$$

Result (type 4, 1123 leaves):

$$\frac{1}{8\,e}\Bigg(i\,b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+8\,i\,b\,\text{ArcCsc}[c\,x]^2-$$
$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]-$$
$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]-$$
$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$
$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\, b\, \pi\, \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$4\, b\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]-$$

$$8\, b\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{i\, \sqrt{e}}{c\, \sqrt{d}}}}{\sqrt{2}}\right]\, \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]-$$

$$2\, b\, \pi\, \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$4\, b\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$8\, b\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{i\, \sqrt{e}}{c\, \sqrt{d}}}}{\sqrt{2}}\right]\, \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]-$$

$$2\, b\, \pi\, \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$4\, b\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$8\, b\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{i\, \sqrt{e}}{c\, \sqrt{d}}}}{\sqrt{2}}\right]\, \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]-$$

$$8\, b\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1-e^{2\, i\, \text{ArcCsc}[c\, x]}\right]+2\, b\, \pi\, \text{Log}\left[\sqrt{e}-\frac{i\, \sqrt{d}}{x}\right]+2\, b\, \pi\, \text{Log}\left[\sqrt{e}+\frac{i\, \sqrt{d}}{x}\right]+$$

$$4\, a\, \text{Log}\left[d+e\, x^2\right]+4\, i\, b\, \text{PolyLog}\left[2,\ \frac{\left(\sqrt{e}-\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$4\, i\, b\, \text{PolyLog}\left[2,\ \frac{\left(-\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$4\, i\, b\, \text{PolyLog}\left[2,\ -\frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+$$

$$\left.4\, i\, b\, \text{PolyLog}\left[2,\ \frac{\left(\sqrt{e}+\sqrt{c^2\, d+e}\right)\, e^{-i\, \text{ArcCsc}[c\, x]}}{c\, \sqrt{d}}\right]+4\, i\, b\, \text{PolyLog}\left[2,\ e^{2\, i\, \text{ArcCsc}[c\, x]}\right]\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 529 leaves, 19 steps):

$$-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$

$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}$$

Result (type 4, 1068 leaves):

$$-\frac{1}{4\,\sqrt{d}\,\sqrt{e}}\,i\,\left(4\,i\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$
$$\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$
$$2\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$
$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$
$$2\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$
$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-$$
$$b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+2\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$
$$2\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$
$$2\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$
$$\left.2\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 101: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{x\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 479 leaves, 19 steps):

$$\frac{i\,(a+b\,\text{ArcCsc}[c\,x])^2}{2\,b\,d}-\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,d}-$$

$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,d}-$$

$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,d}-\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,d}+$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,d}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,d}+$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,d}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,d}$$

Result (type 4, 1089 leaves):

$$-\frac{1}{8\,d}\Bigg(i\,b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+4\,i\,b\,\text{ArcCsc}[c\,x]^2-$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]-$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]-8\,a\,\text{Log}[x]+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{x^2\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 572 leaves, 24 steps):

$$-\frac{b c \sqrt{1-\frac{1}{c^2 x^2}}}{d}-\frac{a}{d x}-\frac{b \operatorname{ArcCsc}[c x]}{d x}-\frac{\sqrt{e}\,(a+b \operatorname{ArcCsc}[c x]) \operatorname{Log}\left[1-\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}+$$

$$\frac{\sqrt{e}\,(a+b \operatorname{ArcCsc}[c x]) \operatorname{Log}\left[1+\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}-$$

$$\frac{\sqrt{e}\,(a+b \operatorname{ArcCsc}[c x]) \operatorname{Log}\left[1-\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}+$$

$$\frac{\sqrt{e}\,(a+b \operatorname{ArcCsc}[c x]) \operatorname{Log}\left[1+\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}-$$

$$\frac{i b \sqrt{e} \operatorname{PolyLog}\left[2,-\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}+\frac{i b \sqrt{e} \operatorname{PolyLog}\left[2,\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}-$$

$$\frac{i b \sqrt{e} \operatorname{PolyLog}\left[2,-\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}+\frac{i b \sqrt{e} \operatorname{PolyLog}\left[2,\frac{i c \sqrt{-d}\, e^{i \operatorname{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 (-d)^{3/2}}$$

Result (type 4, 1241 leaves):

$$-\frac{a}{d x}-\frac{a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e}\, x}{\sqrt{d}}\right]}{d^{3/2}}+$$

$$b \left(-\frac{c \sqrt{1-\frac{1}{c^2 x^2}}\, x+\operatorname{ArcCsc}[c x]}{d x}+\frac{1}{16 d^{3/2}} \sqrt{e} \left(\pi^2-4 \pi \operatorname{ArcCsc}[c x]+8 \operatorname{ArcCsc}[c x]^2-\right.\right.$$

$$32 \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 \operatorname{ArcCsc}[c x])\right]}{\sqrt{c^2 d+e}}\right]+$$

$$4 i \pi \operatorname{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \operatorname{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 i \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \operatorname{ArcCsc}[c x]}}{c \sqrt{d}}\right]+16 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\operatorname{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \operatorname{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 i \pi \operatorname{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \operatorname{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)-$$

$$\frac{1}{16\,d^{3/2}}\sqrt{e}\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\Bigg)\Bigg)$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int\frac{x^5\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 628 leaves, 31 steps):

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2\,c\,e^2}+\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{2\,e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\,\text{ArcCsc}[c\,x]\right)}{2\,e^2}-\frac{b\,d\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]}{2\,e^{5/2}\sqrt{c^2 d+e}}-$$
$$\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{e^3}-\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{e^3}-$$
$$\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{e^3}-\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{e^3}+$$
$$\frac{2\,d\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^3}+\frac{i\,b\,d\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{e^3}+$$
$$\frac{i\,b\,d\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{e^3}+\frac{i\,b\,d\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{e^3}+$$
$$\frac{i\,b\,d\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{e^3}-\frac{i\,b\,d\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^3}$$

Result (type 4, 1604 leaves):

$$\frac{a\,x^2}{2\,e^2}-\frac{a\,d^2}{2\,e^3\left(d+e\,x^2\right)}-\frac{a\,d\,\text{Log}\left[d+e\,x^2\right]}{e^3}+b\left(\frac{x\left(\sqrt{1-\frac{1}{c^2x^2}}+c\,x\,\text{ArcCsc}[c\,x]\right)}{2\,c\,e^2}-\frac{1}{4\,e^{5/2}}\right.$$

$$i\, d^{3/2} \left(-\frac{\text{ArcCsc}[c\, x]}{-i \sqrt{d} \sqrt{e} + e\, x} + \frac{i \left(\frac{\text{ArcSin}\left[\frac{1}{c\, x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(\sqrt{e} + c \left(-i\, c \sqrt{d} - \sqrt{-c^2 d - e} \sqrt{1 - \frac{1}{c^2 x^2}}\right) x\right)}{\sqrt{-c^2 d - e} \left(\sqrt{d} + i \sqrt{e}\, x\right)}\right]}{\sqrt{-c^2 d - e}} \right)}{\sqrt{d}} \right) +$$

$$\frac{1}{4\, e^{5/2}} i\, d^{3/2} \left(-\frac{\text{ArcCsc}[c\, x]}{i \sqrt{d} \sqrt{e} + e\, x} - \frac{i \left(\frac{\text{ArcSin}\left[\frac{1}{c\, x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(-\sqrt{e} + c \left(-i\, c \sqrt{d} + \sqrt{-c^2 d - e} \sqrt{1 - \frac{1}{c^2 x^2}}\right) x\right)}{\sqrt{-c^2 d - e} \left(\sqrt{d} - i \sqrt{e}\, x\right)}\right]}{\sqrt{-c^2 d - e}} \right)}{\sqrt{d}} \right) -$$

$$\frac{1}{8\, e^3} i\, d \Bigg(\pi^2 - 4 \pi\, \text{ArcCsc}[c\, x] + 8\, \text{ArcCsc}[c\, x]^2 -$$

$$32\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(-i\, c \sqrt{d} + \sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi + 2\, \text{ArcCsc}[c\, x]\right)\right]}{\sqrt{c^2 d + e}}\right] +$$

$$4\, i\, \pi\, \text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\, \text{ArcCsc}[c\, x]}}{c \sqrt{d}}\right] -$$

$$8\, i\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\, \text{ArcCsc}[c\, x]}}{c \sqrt{d}}\right] + 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\, \text{ArcCsc}[c\, x]}}{c \sqrt{d}}\right] + 4\, i\, \pi\, \text{Log}\left[1 + \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\, \text{ArcCsc}[c\, x]}}{c \sqrt{d}}\right] -$$

$$8\, i\, \text{ArcCsc}[c\, x]\, \text{Log}\left[1 + \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\, \text{ArcCsc}[c\, x]}}{c \sqrt{d}}\right] - 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+8 i \text{ArcCsc}[c x] \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]-$$

$$4 i \pi \text{Log}\left[\sqrt{e}+\frac{i \sqrt{d}}{x}\right]+8 \text{PolyLog}\left[2, \frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$\left.8 \text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]\right)-$$

$$\frac{1}{8 e^3} i d \left(\pi^2-4 \pi \text{ArcCsc}[c x]+8 \text{ArcCsc}[c x]^2-32 \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(i c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 \text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$4 i \pi \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$16 i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$4 i \pi \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-16 i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+8 i \text{ArcCsc}[c x] \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]-$$

$$4 i \pi \text{Log}\left[\sqrt{e}-\frac{i \sqrt{d}}{x}\right]+8 \text{PolyLog}\left[2, \frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\Bigg)\Bigg)$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 590 leaves, 29 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e\,\left(e+\frac{d}{x^2}\right)}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2\,d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}\right]}{2\,e^{3/2}\,\sqrt{c^2\,d+e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^2}+$$
$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^2}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^2}+$$
$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^2}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^2}-$$
$$\frac{i\,b\,\text{PolyLog}\left[2,\,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^2}-\frac{i\,b\,\text{PolyLog}\left[2,\,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^2}-$$
$$\frac{i\,b\,\text{PolyLog}\left[2,\,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^2}-\frac{i\,b\,\text{PolyLog}\left[2,\,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^2}+\frac{i\,b\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{2\,e^2}$$

Result (type 4, 1442 leaves):

$$\frac{1}{8\,e^2}\Bigg(i\,b\,\pi^2+\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+$$
$$\frac{2\,b\,\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}-i\,\sqrt{e}\,x}+\frac{2\,b\,\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}+i\,\sqrt{e}\,x}+8\,i\,b\,\text{ArcCsc}[c\,x]^2-4\,b\,\text{ArcSin}\left[\frac{1}{c\,x}\right]-$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]-$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]+2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2\,d-e}\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2\,d-e}\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)$$

Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e\left(d+e\,x^2\right)}-\frac{b\,c\,x\,\text{ArcTan}\left[\sqrt{-1+c^2\,x^2}\right]}{2\,d\,e\sqrt{c^2\,x^2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\sqrt{-1+c^2\,x^2}}{\sqrt{c^2\,d+e}}\right]}{2\,d\sqrt{e}\sqrt{c^2\,d+e}\sqrt{c^2\,x^2}}$$

Result (type 3, 286 leaves):

$$-\frac{1}{4e}\left(\frac{2a}{d+ex^2}+\frac{2b\,\text{ArcCsc}[c x]}{d+ex^2}-\frac{2b\,\text{ArcSin}\left[\frac{1}{cx}\right]}{d}+\frac{b\sqrt{e}\,\text{Log}\left[\frac{4 i d e-4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2 d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{d\sqrt{-c^2 d-e}}+\frac{b\sqrt{e}\,\text{Log}\left[\frac{4 i\left(-de+cd\sqrt{e}\left(i c\sqrt{d}+\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{d\sqrt{-c^2 d-e}}\right)$$

Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c x]}{x\left(d+ex^2\right)^2}\,dx$$

Optimal (type 4, 566 leaves, 24 steps):

$$-\frac{e\left(a+b\,\text{ArcCsc}[c x]\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\,\text{ArcCsc}[c x]\right)^2}{2bd^2}+\frac{b\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]}{2d^2\sqrt{c^2 d+e}}-$$

$$\frac{\left(a+b\,\text{ArcCsc}[c x]\right)\text{Log}\left[1-\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2d^2}-\frac{\left(a+b\,\text{ArcCsc}[c x]\right)\text{Log}\left[1+\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2d^2}-$$

$$\frac{\left(a+b\,\text{ArcCsc}[c x]\right)\text{Log}\left[1-\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2d^2}-\frac{\left(a+b\,\text{ArcCsc}[c x]\right)\text{Log}\left[1+\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2d^2}+$$

$$\frac{i b\,\text{PolyLog}\left[2,-\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2d^2}+\frac{i b\,\text{PolyLog}\left[2,\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2d^2}+$$

$$\frac{i b\,\text{PolyLog}\left[2,-\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2d^2}+\frac{i b\,\text{PolyLog}\left[2,\frac{i c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2d^2}$$

Result (type 4, 1408 leaves):

$$\frac{1}{8 d^2}\left(-i b \pi^2+\frac{4 a d}{d+e x^2}+4 i b \pi \text{ArcCsc}[c x]+\right.$$

$$\frac{2 b \sqrt{d}\ \text{ArcCsc}[c x]}{\sqrt{d}-i \sqrt{e}\ x}+\frac{2 b \sqrt{d}\ \text{ArcCsc}[c x]}{\sqrt{d}+i \sqrt{e}\ x}-4 i b \text{ArcCsc}[c x]^2-4 b \text{ArcSin}\left[\frac{1}{c x}\right]+$$

$$16 i b \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 \text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$16 i b \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(i c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 \text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2 b \pi \text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$4 b \text{ArcCsc}[c x] \text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$8 b \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$2 b \pi \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$4 b \text{ArcCsc}[c x] \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$8 b \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$2 b \pi \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$4 b \text{ArcCsc}[c x] \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 b \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-$$
$$2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+8\,a\,\text{Log}[x]+\frac{2\,b\,\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\,\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}+$$
$$\frac{2\,b\,\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\,\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}-$$
$$4\,a\,\text{Log}\left[d+e\,x^2\right]-4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$4\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$\left.4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]\right)$$

Problem 107: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 803 leaves, 51 steps):

$$-\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{x\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}+$$

$$\frac{b\,\text{ArcTanh}\left[\sqrt{1-\frac{1}{c^2\,x^2}}\right]}{c\,e^2}+\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}}\right]}{4\,e^2\,\sqrt{c^2\,d+e}}+$$

$$\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}}\right]}{4\,e^2\,\sqrt{c^2\,d+e}}-\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}$$

Result (type 4, 1634 leaves):

$$\frac{a\,x}{e^2}+\frac{a\,d\,x}{2\,e^2\left(d+e\,x^2\right)}-\frac{3\,a\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,e^{5/2}}+$$

$$b\left(-\frac{1}{4\,e^2}d\left(-\frac{\text{ArcCsc}[c\,x]}{-i\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\,\sqrt{d}-\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right.\right.-$$

$$\frac{1}{4 e^2} d \left(-\frac{\text{ArcCsc}[c\,x]}{i \sqrt{d} \sqrt{e} + e\,x} - \frac{i \left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(-\sqrt{e} + c \left(-i\,c \sqrt{d} + \sqrt{-c^2 d - e} \sqrt{1 - \frac{1}{c^2 x^2}}\right) x\right)}{\sqrt{-c^2 d - e} \left(\sqrt{d} - i \sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d - e}} \right)}{\sqrt{d}} \right) +$$

$$\frac{1}{32 e^{5/2}} 3 \sqrt{d} \Bigg(\pi^2 - 4 \pi\,\text{ArcCsc}[c\,x] + 8\,\text{ArcCsc}[c\,x]^2 -$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(-i\,c \sqrt{d} + \sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi + 2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d + e}}\right] +$$

$$4\,i\,\pi\,\text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] -$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] + 16\,i\,\text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1 + \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] + 4\,i\,\pi\,\text{Log}\left[1 + \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] -$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1 + \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] - 16\,i\,\text{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1 + \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] + 8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1 - e^{2\,i\,\text{ArcCsc}[c\,x]}\right] -$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e} + \frac{i \sqrt{d}}{x}\right] + 8\,\text{PolyLog}\left[2, \frac{\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] +$$

$$8\,\text{PolyLog}\left[2, -\frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c \sqrt{d}}\right] + 4\,\text{PolyLog}\left[2, e^{2\,i\,\text{ArcCsc}[c\,x]}\right] \Bigg) -$$

$$\frac{1}{32\,e^{5/2}}\,3\sqrt{d}\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)+$$

$$\frac{1}{c\,e^2}\left(\frac{1}{2}\,\text{ArcCsc}[c\,x]\,\text{Cot}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]+\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right]-\right.$$

$$\left.\left.\text{Log}\left[\text{Sin}\left[\frac{1}{2}\text{ArcCsc}[c\,x]\right]\right]+\frac{1}{2}\text{ArcCsc}[c\,x]\,\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[c\,x]\right]\right)\right)$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5\,(a+b\,\text{ArcCsc}[c\,x])}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 727 leaves, 33 steps):

$$\frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2 x^2}}}{8\,e^2\,(c^2 d+e)\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\,\text{ArcCsc}[c\,x]}{4\,e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2 x^2}}\,x}\right]}{2\,e^{5/2}\,\sqrt{c^2 d+e}}+$$

$$\frac{b\,(c^2 d+2\,e)\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2 x^2}}\,x}\right]}{8\,e^{5/2}\,(c^2 d+e)^{3/2}}+\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^3}+\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^3}-\frac{(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^3}-$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^3}-\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^3}-$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^3}-\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^3}+\frac{i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{2\,e^3}$$

Result (type 4, 2053 leaves):

$$-\frac{a\,d^2}{4\,e^3\left(d+e\,x^2\right)^2}+\frac{a\,d}{e^3\left(d+e\,x^2\right)}+\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,e^3}+$$

$$b\left(\frac{1}{16\,e^{5/2}}7\,i\,\sqrt{d}\left(-\frac{\text{ArcCsc}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)-\right.$$

$$\frac{1}{16\,e^{5/2}}7\,i\,\sqrt{d}\left(-\frac{\text{ArcCsc}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)-$$

$$\frac{1}{16\,e^{5/2}}d\left(\frac{i\,c\sqrt{e}\,\sqrt{1-\frac{1}{c^2x^2}}\;x}{\sqrt{d}\left(c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsc}[c\,x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\right.$$

$$\left.\frac{i\left(2c^2 d+e\right)\text{Log}\left[\frac{4\,d\sqrt{e}\,\sqrt{c^2 d+e}\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{d\left(c^2 d+e\right)^{3/2}}\right)-\frac{1}{16\,e^{5/2}}$$

$$d\left(-\frac{i\,c\sqrt{e}\,\sqrt{1-\frac{1}{c^2x^2}}\;x}{\sqrt{d}\left(c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsc}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\frac{1}{d\left(c^2 d+e\right)^{3/2}}\right.$$

$$i\left(2c^2 d+e\right)\text{Log}\left[-\left(\left(4\,d\sqrt{e}\,\sqrt{c^2 d+e}\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)\right)\right/\right.$$

$$\left.\left.\left.\left(\left(2 c^2 d+e\right)\left(i \sqrt{d}+\sqrt{e}\, x\right)\right)\right)\right]\right)+\frac{1}{16 e^3} i \left(\pi^2-4 \pi \text{ArcCsc}[c x]+8 \text{ArcCsc}[c x]^2-\right.$$

$$32 \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 \text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$4 i \pi \text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+16 i \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 i \pi \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-16 i \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+8 i \text{ArcCsc}[c x] \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]-$$

$$4 i \pi \text{Log}\left[\sqrt{e}+\frac{i \sqrt{d}}{x}\right]+8 \text{PolyLog}\left[2, \frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$\left.8 \text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]\right)+$$

$$\frac{1}{16 e^3} i \left(\pi^2-4 \pi \text{ArcCsc}[c x]+8 \text{ArcCsc}[c x]^2-32 \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(i c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 \text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$4 i \pi \text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.\left.8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)\right)$$

Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$-\frac{b\,c\,x\,\sqrt{-1+c^2 x^2}}{8\,e\,\left(c^2 d+e\right)\sqrt{c^2 x^2}\,\left(d+e\,x^2\right)}+\frac{x^4\,(a+b\,\text{ArcCsc}[c\,x])}{4\,d\,\left(d+e\,x^2\right)^2}+\frac{b\,c\,\left(c^2 d+2\,e\right)\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8\,d\,e^{3/2}\,\left(c^2 d+e\right)^{3/2}\sqrt{c^2 x^2}}$$

Result (type 3, 390 leaves):

$$\frac{1}{16 e^2}\left(\frac{4 a d}{(d+e x^2)^2}-\frac{8 a}{d+e x^2}-\frac{2 b c e \sqrt{1-\frac{1}{c^2 x^2}}\, x}{(c^2 d+e)(d+e x^2)}-\right.$$

$$\frac{4 b (d+2 e x^2) \text{ArcCsc}[c x]}{(d+e x^2)^2}+\frac{4 b \text{ArcSin}\left[\frac{1}{c x}\right]}{d}+\frac{1}{d (-c^2 d-e)^{3/2}}$$

$$b \sqrt{e} (c^2 d+2 e) \text{Log}\left[\left(16 d \sqrt{-c^2 d-e}\, e^{3/2} \left(i \sqrt{e}+c \left(c \sqrt{d}-i \sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)\right)\Big/\right.$$

$$\left.\left(b (c^2 d+2 e) \left(\sqrt{d}+i \sqrt{e}\, x\right)\right)\right]+\frac{1}{d (-c^2 d-e)^{3/2}}$$

$$b \sqrt{e} (c^2 d+2 e) \text{Log}\left[-\left(\left(16 d \sqrt{-c^2 d-e}\, e^{3/2} \left(-\sqrt{e}+c \left(-i c \sqrt{d}+\sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)\right)\Big/\right.\right.$$

$$\left.\left.\left.\left(b (c^2 d+2 e) \left(i \sqrt{d}+\sqrt{e}\, x\right)\right)\right)\right]\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x (a+b \text{ArcCsc}[c x])}{(d+e x^2)^3}\, dx$$

Optimal (type 3, 193 leaves, 8 steps):

$$\frac{b c x \sqrt{-1+c^2 x^2}}{8 d (c^2 d+e) \sqrt{c^2 x^2} (d+e x^2)}-\frac{a+b \text{ArcCsc}[c x]}{4 e (d+e x^2)^2}-$$

$$\frac{b c x \text{ArcTan}\left[\sqrt{-1+c^2 x^2}\right]}{4 d^2 e \sqrt{c^2 x^2}}+\frac{b c (3 c^2 d+2 e) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8 d^2 \sqrt{e} (c^2 d+e)^{3/2} \sqrt{c^2 x^2}}$$

Result (type 3, 385 leaves):

$$\frac{1}{16}\left(-\frac{4\,a}{e\,(d+e\,x^2)^2}+\frac{2\,b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{d\,(c^2\,d+e)\,(d+e\,x^2)}-\frac{4\,b\,\text{ArcCsc}[c\,x]}{e\,(d+e\,x^2)^2}+\frac{4\,b\,\text{ArcSin}\left[\frac{1}{c\,x}\right]}{d^2\,e}+\right.$$

$$\left(b\,(3\,c^2\,d+2\,e)\,\text{Log}\left[\left(16\,d^2\,\sqrt{-c^2\,d-e}\,\sqrt{e}\,\left(\mathbb{i}\,\sqrt{e}+c\,\left(c\,\sqrt{d}-\mathbb{i}\,\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)\right)\right/\right.$$

$$\left.\left.\left(b\,(3\,c^2\,d+2\,e)\,\left(\sqrt{d}+\mathbb{i}\,\sqrt{e}\,x\right)\right)\right]\right)\Big/\left(d^2\,(-c^2\,d-e)^{3/2}\,\sqrt{e}\right)+\left(b\,(3\,c^2\,d+2\,e)\right.$$

$$\text{Log}\left[-\left(\left(16\,d^2\,\sqrt{-c^2\,d-e}\,\sqrt{e}\,\left(-\sqrt{e}+c\,\left(-\mathbb{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)\right)\right/\right.$$

$$\left.\left.\left.\left(b\,(3\,c^2\,d+2\,e)\,\left(\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x\right)\right)\right)\right]\right)\Big/\left(d^2\,(-c^2\,d-e)^{3/2}\,\sqrt{e}\right)\right)$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{x\,(d+e\,x^2)^3}\,dx$$

Optimal (type 4, 704 leaves, 28 steps):

$$-\frac{b c e \sqrt{1-\frac{1}{c^2 x^2}}}{8 d^2 \left(c^2 d+e\right) \left(e+\frac{d}{x^2}\right) x}+\frac{e^2 \left(a+b \text{ArcCsc}[c x]\right)}{4 d^3 \left(e+\frac{d}{x^2}\right)^2}-\frac{e \left(a+b \text{ArcCsc}[c x]\right)}{d^3 \left(e+\frac{d}{x^2}\right)}+$$

$$\frac{i \left(a+b \text{ArcCsc}[c x]\right)^2}{2 b d^3}+\frac{b \sqrt{e} \text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}\right]}{d^3 \sqrt{c^2 d+e}}-\frac{b \sqrt{e} \left(c^2 d+2 e\right) \text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}\right]}{8 d^3 \left(c^2 d+e\right)^{3/2}}-$$

$$\frac{\left(a+b \text{ArcCsc}[c x]\right) \text{Log}\left[1-\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^3}-\frac{\left(a+b \text{ArcCsc}[c x]\right) \text{Log}\left[1+\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^3}-$$

$$\frac{\left(a+b \text{ArcCsc}[c x]\right) \text{Log}\left[1-\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^3}-\frac{\left(a+b \text{ArcCsc}[c x]\right) \text{Log}\left[1+\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^3}+$$

$$\frac{i b \text{PolyLog}\left[2,-\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^3}+\frac{i b \text{PolyLog}\left[2,\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^3}+$$

$$\frac{i b \text{PolyLog}\left[2,-\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^3}+\frac{i b \text{PolyLog}\left[2,\frac{i c \sqrt{-d} e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^3}$$

Result (type 4, 2114 leaves) :

$$\frac{a}{4 d \left(d+e x^2\right)^2}+\frac{a}{2 d^2 \left(d+e x^2\right)}+\frac{a \text{Log}[x]}{d^3}-\frac{a \text{Log}\left[d+e x^2\right]}{2 d^3}+$$

$$b \left(\frac{1}{16 d^{5/2}} 5 i \sqrt{e} \left(-\frac{\text{ArcCsc}[c x]}{-i \sqrt{d} \sqrt{e}+e x}+\frac{i \left(\frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(\sqrt{e}+c \left(-i c \sqrt{d}-\sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{\sqrt{-c^2 d-e} \left(\sqrt{d}+i \sqrt{e} x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)-\right.$$

$$\frac{1}{16\,d^{5/2}}5\,\mathrm{i}\,\sqrt{e}\left(-\frac{\mathrm{ArcCsc}[c\,x]}{\mathrm{i}\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{\mathrm{i}\left(\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(-\sqrt{e}+c\left(-\mathrm{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}-\mathrm{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{16\,d^2}\sqrt{e}\left(\frac{\mathrm{i}\,c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\left(c^2\,d+e\right)\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsc}[c\,x]}{\sqrt{e}\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\right.$$

$$\left.\frac{\mathrm{i}\left(2\,c^2\,d+e\right)\mathrm{Log}\left[\frac{4\,d\,\sqrt{e}\,\sqrt{c^2\,d+e}\left(\mathrm{i}\,\sqrt{e}+c\left(c\,\sqrt{d}-\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\left(2\,c^2\,d+e\right)\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{d\left(c^2\,d+e\right)^{3/2}}\right)+\frac{1}{16\,d^2}$$

$$\sqrt{e}\left(-\frac{\mathrm{i}\,c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\left(c^2\,d+e\right)\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsc}[c\,x]}{\sqrt{e}\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{1}{d\left(c^2\,d+e\right)^{3/2}}\right.$$

$$\mathrm{i}\left(2\,c^2\,d+e\right)\mathrm{Log}\left[-\left(\left(4\,d\,\sqrt{e}\,\sqrt{c^2\,d+e}\left(-\mathrm{i}\,\sqrt{e}+c\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)\right)\right/\right.$$

$$\left.\left.\left(\left(2\,c^2\,d+e\right)\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)\right)\right)\right]\right)-\frac{1}{16\,d^3}\,\mathrm{i}\left(\pi^2-4\,\pi\,\mathrm{ArcCsc}[c\,x]+8\,\mathrm{ArcCsc}[c\,x]^2-\right.$$

$$32\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(-\mathrm{i}\,c\,\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathrm{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-$$

$$4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)-$$

$$\frac{1}{16\,d^3}\,i\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+8 i \text{ArcCsc}[c x] \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]-$$

$$4 i \pi \text{Log}\left[\sqrt{e}-\frac{i \sqrt{d}}{x}\right]+8 \text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$\left.8 \text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]\right)+\frac{1}{d^3}$$

$$\left.\left(-\text{ArcCsc}[c x] \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]+\frac{1}{2} i \left(\text{ArcCsc}[c x]^2+\text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]\right)\right)\right)$$

Problem 118: Result unnecessarily involves higher level functions.

$$\int x^5 \sqrt{d+e x^2} \left(a+b \text{ArcCsc}[c x]\right) dx$$

Optimal (type 3, 403 leaves, 12 steps):

$$-\frac{b \left(23 c^4 d^2+12 c^2 d e-75 e^2\right) x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{1680 c^5 e^2 \sqrt{c^2 x^2}}-\frac{b \left(29 c^2 d-25 e\right) x \sqrt{-1+c^2 x^2} \left(d+e x^2\right)^{3/2}}{840 c^3 e^2 \sqrt{c^2 x^2}}+\frac{b x \sqrt{-1+c^2 x^2} \left(d+e x^2\right)^{5/2}}{42 c e^2 \sqrt{c^2 x^2}}+$$

$$\frac{d^2 \left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsc}[c x]\right)}{3 e^3}-\frac{2 d \left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsc}[c x]\right)}{5 e^3}+$$

$$\frac{\left(d+e x^2\right)^{7/2} \left(a+b \text{ArcCsc}[c x]\right)}{7 e^3}-\frac{8 b c d^{7/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{105 e^3 \sqrt{c^2 x^2}}+$$

$$\frac{b \left(105 c^6 d^3-35 c^4 d^2 e+63 c^2 d e^2+75 e^3\right) x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{1680 c^6 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 705 leaves):

$$\left(b\, d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \left(\left(105 c^6 d^3 - 35 c^4 d^2 e + 63 c^2 d e^2 + 75 e^3\right) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \left(c^2 d\, \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e\, \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right]\right) + 4\, \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left(\left(35 c^6 d^2 e^2 x^2 - 63 c^4 d e^3 x^2 - 75 c^2 e^4 x^2 + c^8 d^3 \left(128 d - 105 e x^2\right)\right) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + 32 c^8 d^3 x^2 \left(-e\, \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d\, \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/$$

$$\left(840 c^5 e^2 \left(-1 + c^2 x^2\right) \sqrt{d + e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d\, \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e\, \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right]\right) \left(4 d\, \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left(-e\, \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d\, \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right]\right)\right)\right) +$$

$$\frac{1}{1680 c^5 e^3} \sqrt{d + e x^2} \left(16 a c^5 \left(8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6\right) + b\, e \sqrt{1-\frac{1}{c^2 x^2}}\, x \left(75 e^2 + 2 c^2 e \left(19 d + 25 e x^2\right) + c^4 \left(-41 d^2 + 22 d e x^2 + 40 e^2 x^4\right)\right) + 16 b c^5 \left(8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6\right) \text{ArcCsc}[c\, x]\right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{d + e x^2} \left(a + b\, \text{ArcCsc}[c\, x]\right) dx$$

Optimal (type 3, 294 leaves, 11 steps):

$$\frac{b\left(c^2 d+9 e\right) x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{120 c^3 e \sqrt{c^2 x^2}}+\frac{b x \sqrt{-1+c^2 x^2}\left(d+e x^2\right)^{3/2}}{20 c e \sqrt{c^2 x^2}}-\frac{d\left(d+e x^2\right)^{3/2}(a+b \operatorname{ArcCsc}[c x])}{3 e^2}+\frac{\left(d+e x^2\right)^{5/2}(a+b \operatorname{ArcCsc}[c x])}{5 e^2}+\frac{2 b c d^{5/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{15 e^2 \sqrt{c^2 x^2}}-\frac{b\left(15 c^4 d^2-10 c^2 d e-9 e^2\right) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120 c^4 e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 627 leaves):

$$-\left(\left(b d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \left(\left(15 c^4 d^2-10 c^2 d e-9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(10 c^4 d e^2 x^2+9 c^2 e^3 x^2+c^6 d^2 \left(16 d-15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+4 c^6 d^2 x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(60 c^3 e \left(-1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(4 d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{1}{120 c^3 e^2} \sqrt{d+e x^2} \left(8 a c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right)+b e \sqrt{1-\frac{1}{c^2 x^2}}\, x \left(9 e+c^2 \left(7 d+6 e x^2\right)\right)+8 b c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right) \text{ArcCsc}[c x]\right)$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,dx$$

Optimal (type 3, 195 leaves, 9 steps):

$$\frac{b\,x\sqrt{-1+c^2x^2}\,\sqrt{d+e\,x^2}}{6\,c\sqrt{c^2x^2}}+\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e}-\frac{b\,c\,d^{3/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{3\,e\sqrt{c^2x^2}}+\frac{b\left(3\,c^2d+e\right)x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+e\,x^2}}\right]}{6\,c^2\sqrt{e}\,\sqrt{c^2x^2}}$$

Result (type 6, 547 leaves):

$$\left(b\,d\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(\left(3\,c^2d+e\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]\right.\right.$$
$$\left(c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)+$$
$$2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\left(\left(-2\,c^2e^2x^2+2\,c^4d\left(2\,d-3\,e\,x^2\right)\right)\right.$$
$$\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+c^4d\,x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+\right.$$
$$\left.\left.\left.\left.c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(3\,c\left(-1+c^2x^2\right)\sqrt{d+e\,x^2}\left(-4\,c^2e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$
$$\left.c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)$$
$$\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+\right.$$
$$\left.\left.x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{6\,c\,e}\sqrt{d+e\,x^2}\left(b\,e\sqrt{1-\frac{1}{c^2x^2}}\,x+2\,a\,c\left(d+e\,x^2\right)+2\,b\,c\left(d+e\,x^2\right)\text{ArcCsc}[c\,x]\right)$$

Problem 126: Unable to integrate problem.

$$\int\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{x^4}\,dx$$

Optimal (type 4, 328 leaves, 11 steps):

$$-\frac{2\,b\,c\,\left(c^2\,d+2\,e\right)\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{9\,d\,\sqrt{c^2\,x^2}}-\frac{b\,c\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{9\,x^2\,\sqrt{c^2\,x^2}}-\frac{\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,d\,x^3}+\left(2\,b\,c^2\,\left(c^2\,d+2\,e\right)\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],\,-\frac{e}{c^2\,d}\right]\right)\Big/\left(9\,d\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\right)-\left(b\,\left(c^2\,d+e\right)\,\left(2\,c^2\,d+3\,e\right)\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],\,-\frac{e}{c^2\,d}\right]\right)\Big/\left(9\,d\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}\right)$$

Result (type 8, 25 leaves):

$$\int\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{x^4}\,dx$$

Problem 127: Unable to integrate problem.

$$\int\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 4, 453 leaves, 12 steps):

$$-\frac{b\,c\,\left(24\,c^4\,d^2+19\,c^2\,d\,e-31\,e^2\right)\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{225\,d^2\,\sqrt{c^2\,x^2}}-\frac{b\,c\,\left(12\,c^2\,d-e\right)\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{225\,d\,x^2\,\sqrt{c^2\,x^2}}-\frac{b\,c\,\sqrt{-1+c^2\,x^2}\,\left(d+e\,x^2\right)^{3/2}}{25\,d\,x^4\,\sqrt{c^2\,x^2}}-\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,d\,x^5}+\frac{2\,e\,\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{15\,d^2\,x^3}+\left(b\,c^2\,\left(24\,c^4\,d^2+19\,c^2\,d\,e-31\,e^2\right)x\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]\right)\Big/\left(225\,d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\right)-\left(b\,\left(c^2\,d+e\right)\left(24\,c^4\,d^2+7\,c^2\,d\,e-30\,e^2\right)x\,\sqrt{1-c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]\right)\Big/\left(225\,d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}\right)$$

Result (type 8, 25 leaves):

$$\int\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{x^6}\,dx$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int x^3\,\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)dx$$

Optimal (type 3, 374 leaves, 12 steps):

$$-\frac{b\,\left(3\,c^4\,d^2-38\,c^2\,d\,e-25\,e^2\right)x\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{560\,c^5\,e\,\sqrt{c^2\,x^2}}+\frac{b\,\left(13\,c^2\,d+25\,e\right)x\,\sqrt{-1+c^2\,x^2}\,\left(d+e\,x^2\right)^{3/2}}{840\,c^3\,e\,\sqrt{c^2\,x^2}}+\frac{b\,x\,\sqrt{-1+c^2\,x^2}\,\left(d+e\,x^2\right)^{5/2}}{42\,c\,e\,\sqrt{c^2\,x^2}}-\frac{d\,\left(d+e\,x^2\right)^{5/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^2}+\frac{\left(d+e\,x^2\right)^{7/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{7\,e^2}+\frac{2\,b\,c\,d^{7/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{35\,e^2\,\sqrt{c^2\,x^2}}-\frac{b\,\left(35\,c^6\,d^3-35\,c^4\,d^2\,e-63\,c^2\,d\,e^2-25\,e^3\right)x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{560\,c^6\,e^{3/2}\,\sqrt{c^2\,x^2}}$$

Result (type 6, 679 leaves):

$$-\left(\left(\left(b\,d\,\sqrt{1-\frac{1}{c^2 x^2}}\;x^3\left(\left(35\,c^6 d^3-35\,c^4 d^2 e-63\,c^2 d\,e^2-25\,e^3\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]\right.\right.\right.\right.$$

$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]$$

$$\left(\left(35\,c^6 d^2 e^2 x^2+63\,c^4 d\,e^3 x^2+25\,c^2 e^4 x^2+c^8 d^3\left(32\,d-35\,e\,x^2\right)\right)\right.$$

$$\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]+8\,c^8 d^3 x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},\right.\right.$$

$$\left.\left.\left.\left.\left.3,c^2 x^2,-\frac{e\,x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\right)\Big/$$

$$\left(280\,c^5 e\left(-1+c^2 x^2\right)\sqrt{d+e\,x^2}\left(-4\,c^2 e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$

$$\left.c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)$$

$$\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]+x^2\right.$$

$$\left.\left.\left.\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{1}{1680\,c^5 e^2}\sqrt{d+e\,x^2}\left(-48\,a\,c^5\left(2\,d-5\,e\,x^2\right)\left(d+e\,x^2\right)^2+\right.$$

$$b\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\;x$$

$$\left(75\,e^2+2\,c^2 e\left(82\,d+25\,e\,x^2\right)+c^4\left(57\,d^2+106\,d\,e\,x^2+40\,e^2 x^4\right)\right)-$$

$$\left.48\,b\,c^5\left(2\,d-5\,e\,x^2\right)\left(d+e\,x^2\right)^2\text{ArcCsc}[c\,x]\right)$$

Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)\,dx$$

Optimal (type 3, 262 leaves, 10 steps) :

$$\frac{b\left(7 c^2 d+3 e\right) x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{40 c^3 \sqrt{c^2 x^2}}+\frac{b x \sqrt{-1+c^2 x^2}\left(d+e x^2\right)^{3/2}}{20 c \sqrt{c^2 x^2}}+$$

$$\frac{\left(d+e x^2\right)^{5/2}\left(a+b \text{ArcCsc}[c x]\right)}{5 e}-\frac{b c d^{5/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{5 e \sqrt{c^2 x^2}}+$$

$$\frac{b\left(15 c^4 d^2+10 c^2 d e+3 e^2\right) x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{40 c^4 \sqrt{e} \sqrt{c^2 x^2}}$$

Result (type 6, 602 leaves):

$$\left(b d \sqrt{1-\frac{1}{c^2 x^2}} x^3 \left(\left(15 c^4 d^2+10 c^2 d e+3 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$

$$\left(c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$

$$4 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]$$

$$\left(\left(-10 c^4 d e^2 x^2-3 c^2 e^3 x^2+c^6 d^2\left(8 d-15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$

$$2 c^6 d^2 x^2\left(-e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$

$$\left.\left.\left.\left.c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(20 c^3\left(-1+c^2 x^2\right) \sqrt{d+e x^2}\left(-4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.\right.$$

$$\left.c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$

$$\left(4 d \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$

$$\left.\left.x^2\left(-e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$

$$\frac{1}{40 c^3 e} \sqrt{d+e x^2}\left(8 a c^3\left(d+e x^2\right)^2+b e \sqrt{1-\frac{1}{c^2 x^2}} x\left(3 e+c^2\left(9 d+2 e x^2\right)\right)+\right.$$

$$\left.8 b c^3\left(d+e x^2\right)^2 \text{ArcCsc}[c x]\right)$$

Problem 136: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{x^6} \, dx$$

Optimal (type 4, 416 leaves, 12 steps):

$$-\frac{b c \left(8 c^4 d^2+23 c^2 d e+23 e^2\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 d \sqrt{c^2 x^2}}-\frac{4 b c \left(c^2 d+2 e\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 x^2 \sqrt{c^2 x^2}}-\frac{b c \sqrt{-1+c^2 x^2} \left(d+e x^2\right)^{3/2}}{25 x^4 \sqrt{c^2 x^2}}-\frac{\left(d+e x^2\right)^{5/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{5 d x^5}+\left(b c^2 \left(8 c^4 d^2+23 c^2 d e+23 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[c x],-\frac{e}{c^2 d}\right]\right) \Bigg/ \left(75 d \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}\right)-\left(b \left(c^2 d+e\right) \left(8 c^4 d^2+19 c^2 d e+15 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}[c x],-\frac{e}{c^2 d}\right]\right) \Bigg/ \left(75 d \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{x^6} \, dx$$

Problem 137: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \operatorname{ArcCsc}[c x]\right)}{x^8} \, dx$$

Optimal (type 4, 554 leaves, 13 steps):

$$-\frac{1}{3675\, d^2 \sqrt{c^2 x^2}} b\, c\, \left(240\, c^6 d^3 + 528\, c^4 d^2 e + 193\, c^2 d\, e^2 - 247\, e^3\right) \sqrt{-1 + c^2 x^2}\, \sqrt{d + e\, x^2} - \frac{b\, c\, \left(120\, c^4 d^2 + 159\, c^2 d\, e - 37\, e^2\right) \sqrt{-1 + c^2 x^2}\, \sqrt{d + e\, x^2}}{3675\, d\, x^2 \sqrt{c^2 x^2}} - \frac{b\, c\, \left(30\, c^2 d + 11\, e\right) \sqrt{-1 + c^2 x^2}\, \left(d + e\, x^2\right)^{3/2}}{1225\, d\, x^4 \sqrt{c^2 x^2}} - \frac{b\, c\, \sqrt{-1 + c^2 x^2}\, \left(d + e\, x^2\right)^{5/2}}{49\, d\, x^6 \sqrt{c^2 x^2}} - \frac{\left(d + e\, x^2\right)^{5/2} \left(a + b\, \text{ArcCsc}[c\, x]\right)}{7\, d\, x^7} + \frac{2\, e\, \left(d + e\, x^2\right)^{5/2} \left(a + b\, \text{ArcCsc}[c\, x]\right)}{35\, d^2 x^5} + \left(b\, c^2 \left(240\, c^6 d^3 + 528\, c^4 d^2 e + 193\, c^2 d\, e^2 - 247\, e^3\right) x \sqrt{1 - c^2 x^2}\, \sqrt{d + e\, x^2}\ \text{EllipticE}\left[\text{ArcSin}[c\, x], -\frac{e}{c^2 d}\right]\right) \Big/ \left(3675\, d^2 \sqrt{c^2 x^2}\, \sqrt{-1 + c^2 x^2}\, \sqrt{1 + \frac{e\, x^2}{d}}\right) - \left(2\, b\, \left(c^2 d + e\right) \left(120\, c^6 d^3 + 204\, c^4 d^2 e + 17\, c^2 d\, e^2 - 105\, e^3\right) x \sqrt{1 - c^2 x^2}\, \sqrt{1 + \frac{e\, x^2}{d}}\ \text{EllipticF}\left[\text{ArcSin}[c\, x], -\frac{e}{c^2 d}\right]\right) \Big/ \left(3675\, d^2 \sqrt{c^2 x^2}\, \sqrt{-1 + c^2 x^2}\, \sqrt{d + e\, x^2}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{\left(d + e\, x^2\right)^{3/2} \left(a + b\, \text{ArcCsc}[c\, x]\right)}{x^8}\, dx$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a + b\, \text{ArcCsc}[c\, x]\right)}{\sqrt{d + e\, x^2}}\, dx$$

Optimal (type 3, 321 leaves, 11 steps):

$$-\frac{b\, \left(19\, c^2 d - 9\, e\right) x \sqrt{-1 + c^2 x^2}\, \sqrt{d + e\, x^2}}{120\, c^3 e^2 \sqrt{c^2 x^2}} + \frac{b\, x \sqrt{-1 + c^2 x^2}\, \left(d + e\, x^2\right)^{3/2}}{20\, c\, e^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + e\, x^2}\, \left(a + b\, \text{ArcCsc}[c\, x]\right)}{e^3} - \frac{2\, d\, \left(d + e\, x^2\right)^{3/2} \left(a + b\, \text{ArcCsc}[c\, x]\right)}{3\, e^3} + \frac{\left(d + e\, x^2\right)^{5/2} \left(a + b\, \text{ArcCsc}[c\, x]\right)}{5\, e^3} - \frac{8\, b\, c\, d^{5/2}\, x\, \text{ArcTan}\left[\frac{\sqrt{d + e\, x^2}}{\sqrt{d}\, \sqrt{-1 + c^2 x^2}}\right]}{15\, e^3 \sqrt{c^2 x^2}} + \frac{b\, \left(45\, c^4 d^2 - 10\, c^2 d\, e + 9\, e^2\right) x\, \text{ArcTanh}\left[\frac{\sqrt{e}\, \sqrt{-1 + c^2 x^2}}{c\, \sqrt{d + e\, x^2}}\right]}{120\, c^4 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 629 leaves):

$$\left(b\, d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \left(\left(45 c^4 d^2-10 c^2 d e+9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(10 c^4 d e^2 x^2-9 c^2 e^3 x^2+c^6 d^2 \left(64 d-45 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+16 c^6 d^2 x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(60 c^3 e^2 \left(-1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(4 d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+\frac{1}{120 c^3 e^3}\sqrt{d+e x^2} \left(8 a c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right)+b e \sqrt{1-\frac{1}{c^2 x^2}}\, x \left(9 e+c^2 \left(-13 d+6 e x^2\right)\right)+8 b c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right) \text{ArcCsc}[c x]\right)$$

Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\, \text{ArcCsc}[c x]\right)}{\sqrt{d+e x^2}} \, dx$$

Optimal (type 3, 225 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{-1+c^2x^2}\,\sqrt{d+e\,x^2}}{6\,c\,e\,\sqrt{c^2x^2}}-\frac{d\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}+\frac{\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^2}+\frac{2\,b\,c\,d^{3/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{3\,e^2\,\sqrt{c^2x^2}}-\frac{b\,\left(3\,c^2\,d-e\right)\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{6\,c^2\,e^{3/2}\,\sqrt{c^2x^2}}$$

Result (type 6, 554 leaves):

$$-\left(\left(\left(b\,d\,\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\left(\left(3\,c^2\,d-e\right)\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]\,\left(c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)+4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\,\left(\left(c^2\,e^2\,x^2+c^4\,d\,\left(4\,d-3\,e\,x^2\right)\right)\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+c^4\,d\,x^2\,\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/\left(3\,c\,e\,\left(-1+c^2x^2\right)\,\sqrt{d+e\,x^2}\,\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\,\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+x^2\,\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+\frac{1}{6\,c\,e^2}\sqrt{d+e\,x^2}\,\left(-4\,a\,c\,d+b\,e\,\sqrt{1-\frac{1}{c^2x^2}}\,x+2\,a\,c\,e\,x^2+2\,b\,c\,\left(-2\,d+e\,x^2\right)\,\text{ArcCsc}[c\,x]\right)$$

Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 3, 132 leaves, 9 steps):

$$\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{e}-\frac{b c \sqrt{d}\, x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\sqrt{-1+c^2 x^2}}\right]}{e\sqrt{c^2 x^2}}+\frac{b x\,\text{ArcTanh}\left[\frac{\sqrt{e}\sqrt{-1+c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{\sqrt{e}\sqrt{c^2 x^2}}$$

Result (type 6, 271 leaves):

$$-\left(\left(3 b\,(c^2 d+e)\sqrt{1-\frac{1}{c^2 x^2}}\sqrt{d+e x^2}\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]\right)\Big/\left(c e x\left(-3\,(c^2 d+e)\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]+(-1+c^2 x^2)\left(2\,(c^2 d+e)\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]-e\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]\right)\right)\right)\right)+\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{e}$$

Problem 146: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcCsc}[c x]}{x^4\sqrt{d+e x^2}}\,dx$$

Optimal (type 4, 362 leaves, 11 steps):

$$-\frac{b c\,(2 c^2 d-5 e)\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}}{9 d^2\sqrt{c^2 x^2}}-\frac{b c\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}}{9 d x^2\sqrt{c^2 x^2}}-\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{3 d x^3}+\frac{2 e\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{3 d^2 x}+\left(b c^2\,(2 c^2 d-5 e)\,x\sqrt{1-c^2 x^2}\sqrt{d+e x^2}\,\text{EllipticE}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]\right)\Big/\left(9 d^2\sqrt{c^2 x^2}\sqrt{-1+c^2 x^2}\sqrt{1+\frac{e x^2}{d}}\right)-\left(2 b\,(c^2 d-3 e)\,(c^2 d+e)\,x\sqrt{1-c^2 x^2}\sqrt{1+\frac{e x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]\right)\Big/\left(9 d^2\sqrt{c^2 x^2}\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{a+b\,\text{ArcCsc}[c x]}{x^4\sqrt{d+e x^2}}\,dx$$

Problem 147: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{x^5 \left(a + b \operatorname{ArcCsc}[c\, x]\right)}{\left(d + e\, x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 252 leaves, 10 steps):

$$\frac{b\, x \sqrt{-1 + c^2 x^2} \sqrt{d + e\, x^2}}{6\, c\, e^2 \sqrt{c^2 x^2}} - \frac{d^2 \left(a + b \operatorname{ArcCsc}[c\, x]\right)}{e^3 \sqrt{d + e\, x^2}} - \frac{2\, d \sqrt{d + e\, x^2} \left(a + b \operatorname{ArcCsc}[c\, x]\right)}{e^3} + \frac{\left(d + e\, x^2\right)^{3/2} \left(a + b \operatorname{ArcCsc}[c\, x]\right)}{3\, e^3} + \frac{8\, b\, c\, d^{3/2}\, x \operatorname{ArcTan}\left[\frac{\sqrt{d + e\, x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{3\, e^3 \sqrt{c^2 x^2}} - \frac{b \left(9\, c^2 d - e\right) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e\, x^2}}\right]}{6\, c^2 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 586 leaves):

$$-\left(\left(b\,d\,\sqrt{1-\frac{1}{c^2 x^2}}\;x^3\left(\left(9\,c^2\,d-e\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]\right.\right.\right.$$

$$\left(c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\left(\left(c^2\,e^2\,x^2+c^4\,d\left(16\,d-9\,e\,x^2\right)\right)\right.$$

$$\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]+4\,c^4\,d\,x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},\right.\right.$$

$$\left.\left.\left.\left.\left.3,c^2 x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(3\,c\,e^2\left(-1+c^2 x^2\right)\sqrt{d+e\,x^2}\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$

$$\left.c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)$$

$$\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]+x^2\right.$$

$$\left.\left.\left.\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{1}{6\,c\,e^3\sqrt{d+e\,x^2}}\left(b\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\;x\left(d+e\,x^2\right)-\right.$$

$$2$$
$$a$$
$$c$$
$$\left(8\,d^2+4\,d\,e\,x^2-e^2\,x^4\right)-2$$
$$b$$
$$c$$
$$\left(8\,d^2+4\,d\,e\,x^2-e^2\,x^4\right)$$

$$\left.\text{ArcCsc}[c\,x]\right)$$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x^3\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 156 leaves, 9 steps):

$$\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2\sqrt{d+e\,x^2}}+\frac{\sqrt{d+e\,x^2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}-\frac{2\,b\,c\,\sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{e^2\sqrt{c^2\,x^2}}+\frac{b\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{e^{3/2}\sqrt{c^2\,x^2}}$$

Result (type 6, 326 leaves):

$$\left(2\,b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x^3\left(-\left(\left(2\,c^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)+\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2\,x^2,-\frac{e\,x^2}{d}\right]\Big/\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/\left(e\left(-1+c^2\,x^2\right)\sqrt{d+e\,x^2}\right)+\frac{\left(2\,d+e\,x^2\right)\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2\sqrt{d+e\,x^2}}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{e\sqrt{d+e\,x^2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{\sqrt{d}\,e\sqrt{c^2\,x^2}}$$

Result (type 6, 190 leaves):

$$\left(2\,b\,c^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(\left(-1+c^2\,x^2\right)\sqrt{d+e\,x^2}\right.$$
$$\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+\right.$$
$$\left.\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)-\frac{a+b\,\text{ArcCsc}[c\,x]}{e\,\sqrt{d+e\,x^2}}$$

Problem 155: Unable to integrate problem.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{x^2\,\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 275 leaves, 10 steps):

$$-\frac{b\,c\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{d^2\,\sqrt{c^2\,x^2}}-\frac{a+b\,\text{ArcCsc}[c\,x]}{d\,x\,\sqrt{d+e\,x^2}}-\frac{2\,e\,x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{d^2\,\sqrt{d+e\,x^2}}+$$
$$\frac{b\,c^2\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]}{d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}}-$$
$$\frac{b\,\left(c^2\,d+2\,e\right)\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]}{d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}$$

Result (type 8, 25 leaves):

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{x^2\,\left(d+e\,x^2\right)^{3/2}}\,dx$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int\frac{x^5\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 243 leaves, 10 steps):

$$\frac{b\,c\,d\,x\,\sqrt{-1+c^2\,x^2}}{3\,e^2\,\left(c^2\,d+e\right)\,\sqrt{c^2\,x^2}\,\sqrt{d+e\,x^2}}-\frac{d^2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^3\,\left(d+e\,x^2\right)^{3/2}}+\frac{2\,d\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^3\,\sqrt{d+e\,x^2}}+$$
$$\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^3}-\frac{8\,b\,c\,\sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{3\,e^3\,\sqrt{c^2\,x^2}}+\frac{b\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{e^{5/2}\,\sqrt{c^2\,x^2}}$$

Result (type 6, 416 leaves):

$$\left(2\,b\,c\,d\,\sqrt{1-\frac{1}{c^2 x^2}}\,x^3\right.$$

$$\left(-\left(\left(8\,c^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},\right.\right.\right.\right.$$

$$2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]+$$

$$\left.\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\right]\right)\right)+\left(3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\Big/$$

$$\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]+\right.\right.$$

$$\left.\left.\left.\left.c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(3\,e^2\left(-1+c^2 x^2\right)\sqrt{d+e\,x^2}\right)+\left(b\,c\,d\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\left(d+e\,x^2\right)+\right.$$

$$a\left(c^2 d+e\right)\left(8\,d^2+12\,d\,e\,x^2+3\,e^2 x^4\right)+$$

$$\left.b\left(c^2 d+e\right)\left(8\,d^2+12\,d\,e\,x^2+3\,e^2 x^4\right)\text{ArcCsc}[c\,x]\right)\Big/\left(3\right.$$

$$\left.e^3\left(c^2 d+e\right)\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int\frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$-\frac{b\,c\,x\,\sqrt{-1+c^2 x^2}}{3\,e\left(c^2 d+e\right)\sqrt{c^2 x^2}\,\sqrt{d+e\,x^2}}+\frac{d\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e^2\left(d+e\,x^2\right)^{3/2}}-$$

$$\frac{a+b\,\text{ArcCsc}[c\,x]}{e^2\sqrt{d+e\,x^2}}+\frac{2\,b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\right]}{3\,\sqrt{d}\,e^2\,\sqrt{c^2 x^2}}$$

Result (type 6, 270 leaves):

$$\left(4\,b\,c^3\sqrt{1-\frac{1}{c^2\,x^2}}\;x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(3\,e\,\left(-1+c^2\,x^2\right)\sqrt{d+e\,x^2}\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)+\left(-b\,c\,e\sqrt{1-\frac{1}{c^2\,x^2}}\;x\left(d+e\,x^2\right)-a\left(c^2\,d+e\right)\left(2\,d+3\,e\,x^2\right)-b\left(c^2\,d+e\right)\left(2\,d+3\,e\,x^2\right)\text{ArcCsc}[c\,x]\right)\Big/\left(3\,e^2\left(c^2\,d+e\right)\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int\frac{x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{b\,c\,x\sqrt{-1+c^2\,x^2}}{3\,d\left(c^2\,d+e\right)\sqrt{c^2\,x^2}\sqrt{d+e\,x^2}}-\frac{a+b\,\text{ArcCsc}[c\,x]}{3\,e\left(d+e\,x^2\right)^{3/2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{3\,d^{3/2}\,e\sqrt{c^2\,x^2}}$$

Result (type 6, 254 leaves):

$$\left(2\,b\,c^3\sqrt{1-\frac{1}{c^2\,x^2}}\;x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(3\,d\,\left(-1+c^2\,x^2\right)\sqrt{d+e\,x^2}\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)+\left(-a\,d\left(c^2\,d+e\right)+b\,c\,e\sqrt{1-\frac{1}{c^2\,x^2}}\;x\left(d+e\,x^2\right)-b\,d\left(c^2\,d+e\right)\text{ArcCsc}[c\,x]\right)\Big/\left(3\,d\,e\left(c^2\,d+e\right)\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 164: Unable to integrate problem.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 296 leaves, 10 steps):

$$-\frac{b\,c\,e\,x^2\,\sqrt{-1+c^2\,x^2}}{3\,d^2\,\left(c^2\,d+e\right)\,\sqrt{c^2\,x^2}\,\sqrt{d+e\,x^2}}+\frac{x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,d\,\left(d+e\,x^2\right)^{3/2}}+$$

$$\frac{2\,x\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,d^2\,\sqrt{d+e\,x^2}}+\frac{b\,c^2\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]}{3\,d^2\,\left(c^2\,d+e\right)\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}}+$$

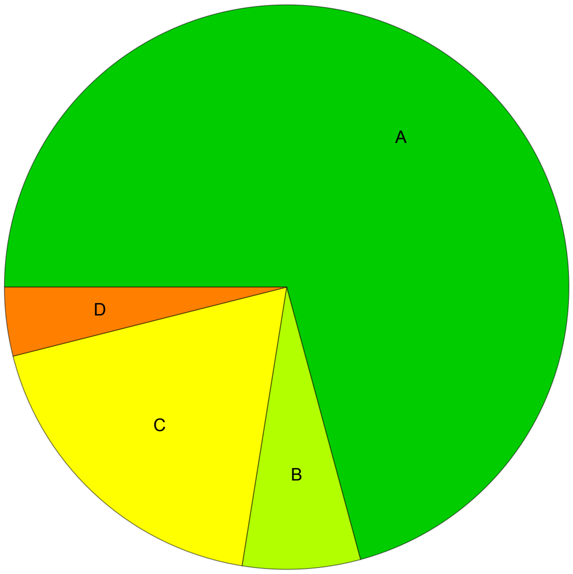
$$\frac{2\,b\,x\,\sqrt{1-c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]}{3\,d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}$$

Result (type 8, 22 leaves) :

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Summary of Integration Test Results

178 integration problems



A - 126 optimal antiderivatives

B - 12 more than twice size of optimal antiderivatives

C - 33 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 0 integration timeouts